Theory of the tunneling resonances of the bilayer electron systems in strong magnetic field

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We develop a theory for the anomalous interlayer conductance peaks observed in bilayer electron systems at $\nu = 1$. Our model shows the that the size of the peak at zero bias decreases rapidly with increasing in-plane magnetic field, but its location is unchanged. The I-V characteristic is linear at small voltages, in agreement with experimental observations. In addition we make quantitative predictions for how the inter-layer conductance peaks vary in position with in-plane magnetic field at high voltages. Finally, we predict novel bi-stable behavior at intermediate voltages.

The discovery of dual peaks in the interlayer tunneling conductance in double layer quantum Hall systems at total filling factor $\nu = 1$ by Spielman *et. al.* [1] has stimulated a number of theoretical studies.[2, 3, 4, 5, 6, 7] Such interlayer tunneling measurements are a valuable tool to study the dynamical aspects of bilayer electron systems (BLES), since the incident quasiparticles interact with the tunneling barrier, impurities, interface roughness of the wells, as well as the two-dimensional electron system (2DES) in the wells. The last of these gives rise to an inelastic scattering mechanism where the tunneling quasiparticle excites the collective modes of the BLES in strong magnetic field. The dispersion relations of these modes, and hence the inelastic scattering rate are sensitive to the external in-plane field, producing a resonant peak in the conductance which varies in voltage with the applied in-plane magnetic field. The observation of this peak at non-zero voltage was been reported by Spielman *et al.* [1]. In addition, tunneling quasiparticles can interact with topological defects in the order parameter such as merons (which carry the electrical charge) causing phase decoherence and dissipation in the tunneling current [3], with an inelastic scattering rate denoted by $\alpha_\perp$. The height and the width of the interlayer current peak is limited by such dissipative effects.

The least understood aspect of the experiment is the yet unexplained peak in the conductance at zero voltage and its dependence on the in-plane magnetic field. The location of the zero bias voltage peak is insensitive to the in-plane magnetic field, but the height of this peak decreases rapidly with in-plane magnetic field. Among the many theories of this experiment[2, 3, 4, 5, 6] there are different and even controversial interpretations for these observations. For example in Refs. [2, 5], it has been argued that the conductance peak is the remnant of the long Josephson effect, however, in Ref. [6] the microscopic calculation indicates this system can be described by the excitonic superfluids.

In this paper we propose an approach based on a damped Landau-Lifshitz equation for the pseudospin order parameter. Our model differs from other approaches[6] in the types of dissipation included and how interlayer current is calculated. [2, 3, 4, 5, 6] These differences permit allow the model to capture previously unexplained features of the interlayer tunneling spectrum in low and high bias voltages. Our model reproduces the experimentally observed linear interlayer current ($I_t \sim V$) at low voltages (fig. 1), along with the value of the peak in the conductance at zero bias. Taking the intrinsic damping mechanism into account, we show the height of the conductance peak falls off as $1/Q^2$, where $Q \equiv (B_\parallel/B_\perp)(d/\ell_0^2)$ is the in-plane magnetic field wave vector, ℓ_0 is the magnetic length, and $B_\parallel$ ($B_\perp$) is the component of the the field parallel (perpendicular) to the quantum well. At higher bias voltages the system enters a non-linear regime with a conductance peak whose position changes with in-plane magnetic field. At still higher voltages, the current decays $I_t \sim 1/V^3$. Furthermore, we argue the low voltage state is different from the Josephson effect, and therefore argue against possibility of the Josephson effect at $V \to 0$. Moreover, we predict the existence of a new bistable state between a "rotating" and "locked" states for the order parameter for small voltages in the presence of in-plane magnetic field, could be realized, depending upon the initial conditions.

Our interpretation of the electrical current differs from other approaches. We model the steady state flow of quasiparticles, as an *imperfect capacitor* with a non-linear charging energy, hence the number of electrons and holes in different layers is fixed (but not equal). This system of a parallel resistor and capacitor (RC), connected to an external electrochemical potential gives the steady state dissipative interlayer current $I = \delta q(V)/\tau_z$. Here δq is the restored charge in the capacitor (which is a function of external potential V), and $\tau_z = RC$ is the relaxation time of the circuit. This analogy leads us to introduce a damping coefficient $\alpha_z (\equiv 1/\tau_z)$ in the theory of the interlayer tunneling effect. This coefficient, absent in previous
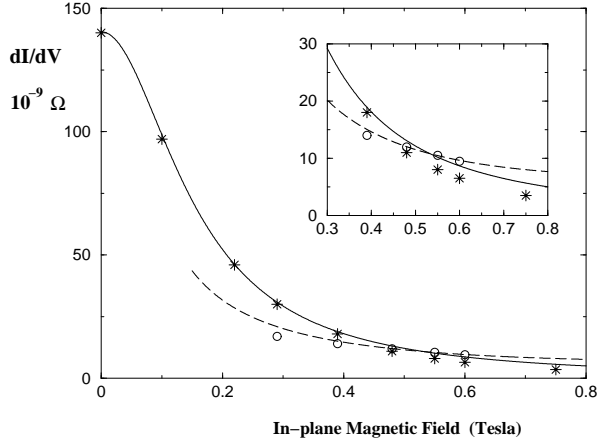
FIG. 1: The height of the zero voltage conductance peak (solid line) and the conductance peak at non-zero voltage (dashed line) versus the in-plane magnetic field, obtained by Eqs. (9) and (7). Stars and circles are the experimental data for total electron density $N_T = 6.0 \times 10^{10} cm^{-2}$. Inset: the magnified crossing region of the main panel. The height of the zero-voltage peak falls off as $1/Q^2$; the non-zero voltage peak varies as $1/Q$.

models, is crucial to producing a zero bias peak.

The low energy physics of the bilayer quantum Hall (pseudo) ferromagnets, which is based upon the microscopic Hartree-Fock model at odd integer filling fractions [8, 9], is described by the following effective Hamiltonian:

$$\mathcal{H} = -\frac{eV}{2}m_z + \frac{\rho_E}{2}\left\{ \left(\frac{\partial m_x}{\partial x}\right)^2 + \left(\frac{\partial m_y}{\partial x}\right)^2 \right\} + \beta m_z^2$$
$$- \Delta_{SAS}\Big\{ m_x\cos(Qx) + m_y\sin(Qx) \Big\}, \qquad (1)$$

where $\hat{m}(x,t)$ is the order parameter unit vector (m_z is the particle density difference between two layers, and m_x, and m_y are its canonical conjugate variables), ρ_E is the in-plane (pseudo)spin stiffness, β gives a hard axis anisotropy due to the capacitance energy cost, Δ_{SAS} is the tunneling amplitude, V is the external interlayer bias voltage, and Q is defined above.

Without the external leads, $m_z = 0$ is the lowest energy state of an isolated BLES. Connecting the layers to the external leads bring this system out of equilibrium. Similar to an imperfect capacitor, the quasi-particles can flow between the layers, via a leakage current. Before reaching steady state, the displacement current dm_z/dt, (which passes through the capacitor even at $\Delta_{SAS} = 0$), can be measured. At steady state, the charge density on the capacitor becomes fixed, even though there is still a leakage current. The capacitance charge is given by $< m_z(x,t) >$ ($<>$ is the average over the temporal and the spatial fluctuations), which is fixed and hence $< dm_z/dt >= 0$. However, there is still a steady state current due to the interlayer quasiparticle tunneling channel. (This result is distinct from other approaches in

which $< dm_z/dt >= 0$ would imply there is no tunneling current.)

The energy loss by dissipative quasiparticle tunneling can be given phenomenologically by including a damping coefficient α_z, coupled to m_z.

$$I_t = \alpha_z e < m_z(x,t) > . \qquad (2)$$

The parameter α_z^{-1} which controls the resistance of the system is equivalte to the RC relaxation rate, and scales like $1/\sqrt{\beta\Delta_{SAS}}$. To calculate m_z we start from the *damped* Landau-Lifshitz equations (see e.g. [11]):

$$\vec{R} + \frac{\partial\hat{m}}{\partial t} = \left(\hat{m} \times \vec{H}_{eff}\right). \qquad (3)$$

$\vec{H}_{\text{eff}} = -2\left\{ \frac{\partial\mathcal{H}}{\partial\hat{m}} - \frac{\partial}{\partial x}\left[\frac{\partial\mathcal{H}}{\partial\frac{\partial\hat{m}}{\partial x}}\right] \right\}$, is the effective magnetic field, $\vec{R}$ is a Landau-Lifshitz damping term (and $\hbar = 1$). Our phenomenological model for the damping in BLES are characterized by two coefficients α_z, and $\alpha_\perp$ that define the damping vector $\vec{R}$:

$$R_x = -\alpha_z m_z^2 m_x^0 - \alpha_\perp m_y(m_y m_x^0 - m_x m_y^0),$$
$$R_y = -\alpha_z m_z^2 m_y^0 + \alpha_\perp m_x(m_y m_x^0 - m_x m_y^0),$$
$$R_z = \alpha_z m_z(m_x m_x^0 + m_y m_y^0). \qquad (4)$$

which is defined so that $\hat{m} \cdot \vec{R} = 0$ and thus the length of $\hat{m}$ is conserved ($\hat{m} \cdot \partial_t \hat{m} = 0$). The vector $\hat{m}^0$ is the equilibrium value of the order parameter. It is important to note the solutions of these Landau-Lifshitz equations exhibit different behavior as the bias voltage is increased. As $V \to 0$ (but $\Delta_{SAS} \neq 0$) the tunneling term is dominant and the order parameter stays (almost) along the x-direction. Without damping the order parameter can precess around this direction, tracing out a cone centered on the m_x axis. The effect of the damping is to equilibrate the order parameter along the x-direction in a finite time, hence we assume $\hat{m}^0 = (1,0,0)$ in Eq.(4). Increasing the (small) bias voltage V alters the equilibrium state. Without damping, the lowest energy state can be determined by minimizing Hamiltonian (1). The role of damping is to relax the excited states to some steady state $\hat{m}^0$. As V increases, the direction of $\hat{m}^0$ rotates towards the y-axis and tilts up slightly from xy-plane. The non-zero value of m_y^0 reflects a non-zero Josephson current, but this current will vanish if $V = 0$ due to the damping. We refer to this family of solutions as "damping-locked states." Starting from $\hat{m}^0 = (1,0,0)$ in Eq.(4), and cranking up the bias voltages, when $eV \approx \sqrt{8\beta\Delta_{SAS}}$, the electrostatic energy becomes comparable to the tunneling energy, and the order parameter starts to precess around a direction given by $\vec{H}_{\text{eff}}$. The system can no longer follow the tunneling term, and the order parameter becomes "unlocked" due to the bias voltage. The amplitude of these "unlocked" oscillatory solutions decreases with increasing V, so that at very large bias voltages the order
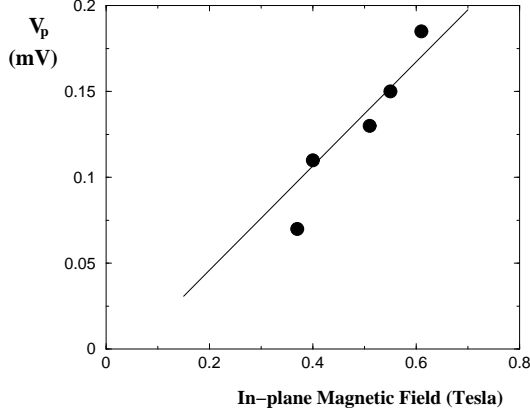
FIG. 2: Locations of non-zero voltage conductance peaks versus in-plane magnetic field. The theoretical curve (solid line) is derived from expression (7), while the filled circles are experimental data [7] for the same total electron density $N_T = 6.0 \times 10^{10} cm^{-2}$.

parameter aligns with the z-axis as $V \to \infty$. In the rest of the paper we detail the solutions of the damped Landau-Lifshitz equations, and evaluate the interlayer current. Starting from the uniform solution m^0, it is straightforward to linearize the Landau-Lifshitz equations, to find their inhomogeneous solutions, using a starting point in our perturbative expansion that is different in low and high V limits (due to different nature of solutions).

Large voltages: At high voltages ($eV \gg \sqrt{8\beta\Delta_{SAS}}$) the pseudospin rotates around z-axis with a frequency $\omega \equiv eV/\hbar$. It is then more convenient to work in a rotating frame which can be introduced by the transformation $n_\perp \equiv m_\perp \exp(i\omega t)$, $n_z \equiv m_z$, and choosing $n^{(0)} = (1,0,0)$ which is equivalent to $m^{(0)} = (\cos\omega t, \sin\omega t, 0)$ in the rest frame. Following this, the Hamiltonian (1) in the rotating frame can be transformed to [8, 9]:

$$\mathcal{H} = \frac{\rho_E}{2}\left\{\left(\frac{\partial n_x}{\partial x}\right)^2 + \left(\frac{\partial n_y}{\partial x}\right)^2\right\} + \beta(n_z)^2$$
$$-\Delta_{SAS}\left\{n_x\cos(\omega t + Qx) + n_y\sin(\omega t + Qx)\right\}. \quad (5)$$

Replacing $\hat{n}^0 = (1,0,0)$ in Eq.(4), the Landau-Lifshitz equations can be derived:

$$-\alpha_\perp n_y^2 - \alpha_z n_z^2 + \frac{\partial n_x}{\partial t} = -4\beta n_y n_z - 2\rho_E n_z \frac{\partial^2 n_y}{\partial x^2} - 2\Delta_{SAS}n_z\sin(\omega t + Qx)$$

$$\alpha_\perp n_y n_x + \frac{\partial n_y}{\partial t} = 4\beta n_x n_z + 2\rho_E n_z \frac{\partial^2 n_x}{\partial x^2} + 2\Delta_{SAS}n_z\cos(\omega t + Qx)$$

$$\alpha_z n_z n_x + \frac{\partial n_z}{\partial t} = 2\rho_E\left\{n_x\frac{\partial^2 n_y}{\partial x^2} - n_y\frac{\partial^2 n_x}{\partial x^2}\right\} + 2\Delta_{SAS}\left\{n_x\sin(\omega t + Qx) - n_y\cos(\omega t + Qx)\right\}. \quad (6)$$

The last equation in Eqs. (6) can be interpreted as the continuity equation for the interlayer current. The external (but self-consistent) chemical potential contributes to the current via the first term in the left side of this equation ($n_x \approx 1$). The first term in the right hand side gives the current density due to phase slips, $J = \rho_E \partial\varphi(x)/\partial x$, equivalent to a dissipationless supercurrent density of the excitonic condensation. In the presence of the small in-plane magnetic field (the commensurate state) $J = \rho_E Q$. Finally, the last term is analogous to the AC Josephson current. The perturbative solution around $\hat{n}^0 = (1,0,0)$ can be achieved by making the harmonic expansion: $\hat{n} = \vec{A}\sin(\omega t + Qx) + \vec{B}\cos(\omega t + Qx) + \vec{n}^0 + \cdots$. Substituting this into equations (6), A and B can be determined after linearizing the Landau-Lifshitz equation, and one can derive the non-homogeneous leading terms in $\hat{n}(x,t)$. Plugging this into Eq.(2) (after replacing the coefficients A and B in $\hat{n}(x,t)$), and making the space-time average, we finally end up with an expression for the steady state

tunneling DC current

$$I_t = \frac{8e\beta\Delta_{SAS}^2\omega(\alpha_z + \alpha_\perp)}{(8\beta\rho_E Q^2 - \omega^2 + \alpha_z\alpha_\perp)^2 + \omega^2(\alpha_z + \alpha_\perp)^2}. \quad (7)$$

Solution (7) qualitatively well describes the peaks at $\omega \approx \sqrt{8\beta\rho_E}Q$, corresponding to the resonance condition for the gapless acoustic mode. The height and the width of the interlayer current are controlled by the damping. In the absence of the damping $I_t (\propto \Delta_{SAS}^2)$ has a singular peak at $\sqrt{8\beta\rho_E}Q$. Solution (7) is parametrically unstable for $\omega^2 \geq 8\beta\rho_E Q^2$ ("tachionic" regime) as in the case of long Josephson junctions, (see e.g. [12]), but it is stable for the large voltage limit $\omega^2 \gg 8\beta\rho_E Q^2$ where the I-V characteristic follows the power law $I_t \sim 1/V^3$. Although it has been speculated by Fogler and Wilczek [5] that the interlayer current peaks resemble the long AC Josephson effect [10] (where the location of the peaks are shifted by $\alpha_\perp$, and α_z), here we argue the observation of these peaks is the manifestation of the spontaneous phase coherence, where the lowest energy state of the electrons is in a symmetric linear combination of two layers which allow the

electrons tunnel through the energy barrier between two quantum wells without resistance (if $\alpha_\perp = \alpha_z = 0$). It is also possible to search for the excitonic superfluid modes through out the interlayer tunneling measurement. The staggered supercurrent density of the excitonic pairs in the superfluid state is given by $J_s = \rho_E Q_s$ for low tunneling energies. The velocity of the collective modes alters by J_s: $\omega_Q \rightarrow \omega_{Q+Q_s}$. In the interlayer tunneling effect, the incident electrons are scattered by these collective modes. From the conservation of the energy and momentum, we have $eV = \hbar\omega_{Q+Q_s}$, i.e., the superfluid current shifts the location of the peaks.

Close to the order-disorder transition point, the experiment suggests the possibility of the coexistence of the incompressible state with the compressible state [13]. In this circumstances, the current from one layer to other layer can transfer through the phase coherent channel, as was described above, and also through the quasiparticle channels which are in the "uncorrelated" state. The contribution from the latter to the interlayer current in low voltages is dominant, which gives rise to a linear I-V characteristic, and therefore the possiblity of the DC Josephson effect is ruled out (see below).

Small Voltages: For low bias voltages, we begin with the Landau-Lifshitz equations, Eqs.(3), and (4) (in the rest frame). Similar to Eqs.(6) we can derive a set of equations in this limit. The perturbative solution about $\hat{m} = (1,0,0)$ can be obtained by a calculation similar to that above. First we consider the simplest case of the zero in-plane magnetic field ($Q = 0$). The uniform and static solution can be obtained easily. Given these solutions, one can find the interlayer conductance

$$G_t = \frac{2e^2\alpha_z\Delta_{SAS}}{4\Delta_{SAS}(2\beta + \Delta_{SAS}) + \alpha_z\alpha_\perp}. \qquad (8)$$

A similar technique can be used to derive the analytical (non-uniform) solution in the presence of in-plane magnetic field if $\rho_E Q^2 \gg \Delta_{SAS}$. The effect of the tunneling term (which is similar to the driving force) is to create only the harmonics with "wave-number" Q which itself will nonlinearly generate zero, $2Q$ and higher harmonics. Because of the damping, the amplitudes of all other harmonics (but the first harmonic) is expected to be zero in the zero temperature limit. It is therefore natural to start with the following harmonic expansion in the rest frame $m_x = 1 - m^+m^-/2$, where $m^+ = Ae^{iQx} + Be^{-iQx} + m_0^+ + \cdots$, and $m^+ = m_z + im_y$. Assuming that the leading perturbative terms should be the first harmonics of the driven wave-number we can determine $A = -B = \Delta_{SAS}(4\beta - i\alpha_\perp)/(8\beta\rho_E Q^2 + \alpha_z\alpha_\perp)$. Substituting these into the Landau-Lifshitz equations, and linearizing them in terms of A, and B, we can find the zeroth harmonic term m_0^+, and then the interlayer DC current $I_t = \alpha_z e < m_z(x,t) >= \alpha_z em_{z0}$, and the

interlayer conductance can be obtained

$$G_t = \frac{8e^2\beta\Delta_{SAS}^2\alpha_z}{\alpha_z\alpha_\perp(8\beta\rho_E Q^2 + \alpha_z\alpha_\perp) + (32\beta^2 + 2\alpha_\perp^2)\Delta_{SAS}^2}. \quad (9)$$

The height of this peak falls off like $1/Q^2$ (for high in-plane magnetic field), but the location of its center does not vary with in-plane magnetic field, consistent with [1] (see also Fig. 1). This perturbative solution is valid for small V's and large Q, and it coincides with the residual zero voltage peak in the presence of in-plane magnetic field.

We note in passing that these solutions are valid in their respective limits, but that at intermediate voltages it may be possible to have more than one solution to a nonlinear differential equation. The basin of attraction of the solutions will depend upon damping and other details of the system.

Numerical results: Our estimate shows the cross over between low and high bias voltages occurs at $0.01mV$. To examine the accuracy of our model, the interlayer conductance peaks have been drawn, by using two adjustable parameters. In Figs. 1 and 2 the fit to the experimental data [1] is obtained by the following damping coefficients: $\alpha_\perp = 0.25\alpha_z$, $\alpha_\perp\alpha_z(\alpha_\perp + \alpha_z)^2 = 32\beta^2\Delta_{SAS}^2$ (for $\Delta_{SAS} = 90\ \mu K$, and $\sqrt{8\beta\Delta_{SAS}} = 70mK$, we find $\alpha_z = 75mK$, and $\alpha_\perp = 18mK$). In Fig. 1, the height of the central residual (solid line) and the split off peaks (dashed line) vs. $B_\parallel$ have been derived by Eq. (7), and (9). In Fig. 2 we present the locations of the split off peaks derived from Eq. (7).

We presented a physical picture based on a driven-damped easy-plane pseudospin ferromagnet model for the experimental observation of the interlayer conductance peak in a bi-layer electron system at $\nu = 1$ [1]. The first theoretical prediction for low bias voltage conductance peak vs. in-plane magnetic field has been made. It has been shown, at high voltages, due to the non-linear behavior of the capacitance energy, the inter-layer current shifts by in-plane magnetic field.

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